Cosmological Constraints on Newton's Gravitational Constant for Matter and Dark Matter

Jordi Salvado

In collaboration with **Yang Bai** and **Ben Stefanek**

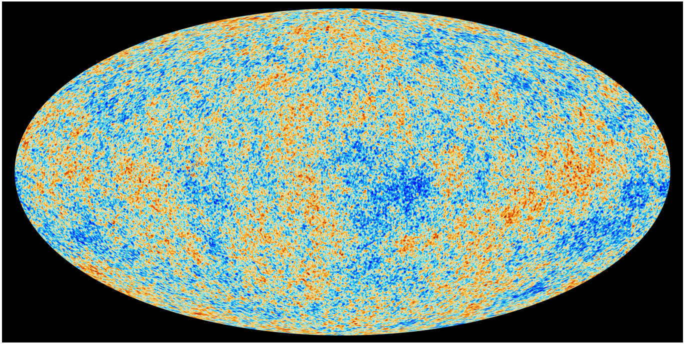
Wisconsin IceCube Particle Astrophysics Center (WIPAC) and

Department of Physics University of Wisconsin

Fermilab February, 26 2015

Table of Contents

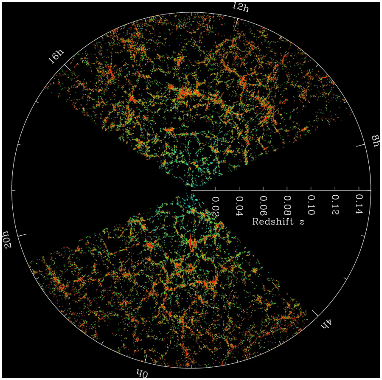
Introduction



Planck $z \approx 1100 \quad \frac{\delta T}{T} \approx 10^{-5}$

$$\left\langle \frac{\delta T}{\bar{T}}(\hat{n})\frac{\delta T}{\bar{T}}(\hat{n}') \right\rangle = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\hat{n}\cdot\hat{n}')$$

Introduction



SDSS $z \approx 0.1$ $\frac{\delta\rho}{\rho}$ is small only at large scales.

$$P_{gal}(k, z) = \langle | \frac{\delta\rho(k_i, z)}{\rho} |^2 \rangle$$

$\delta\rho(k_i, z)$ is the F.T. of the density fluctuations.

Introduction

- The metric for a general homogeneous and isotropic Universe is,

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2\right),$$

  all the dynamics is in the function $a(t)$, $a(t)$ and K are determined by the content of the Universe

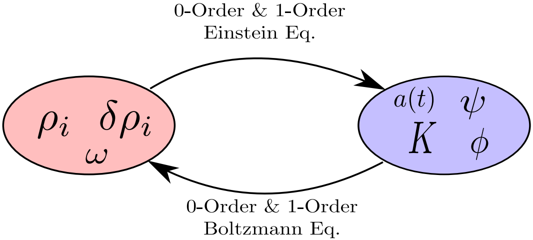
- To take into account the small deviations we need to go beyond the homogeneous and isotropic solution. F.e for scalar perturbations

$$ds^2 = (1 - 2\psi(t,x))dt^2 - (1 + 2\phi(t,x))a^2(t)\left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2\right).$$

Introduction

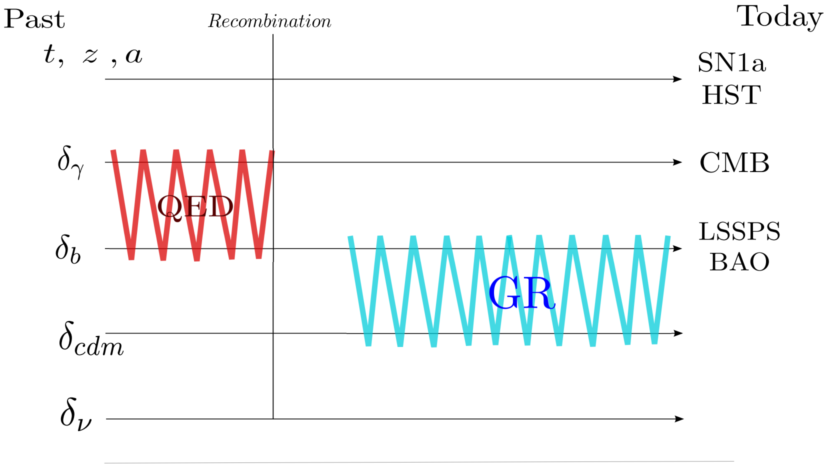
Type	0-order	1-order
Matter	$\rho_{cdm}(t),\ \rho_b(t)$	$\delta\rho_{cdm}(t,x),\ \delta\rho_b(t,x)$
Radiation	$\rho_\gamma(t),\ \rho_{N_{rel}}(t)$	$\delta\rho_\gamma(t,x),\ \delta\rho_\nu(t,x)$
Dark energy	$\rho_\Lambda(t),\ \omega$	

- Matter and radiation evolution is determined by Boltzmann equations up to first order in $\delta\rho_i/\rho_i$.
- Geometry is determined by Einstein equations $H(z) = \sqrt{\sum_i \rho_i(z)}$
- Both sets of eqs are coupled



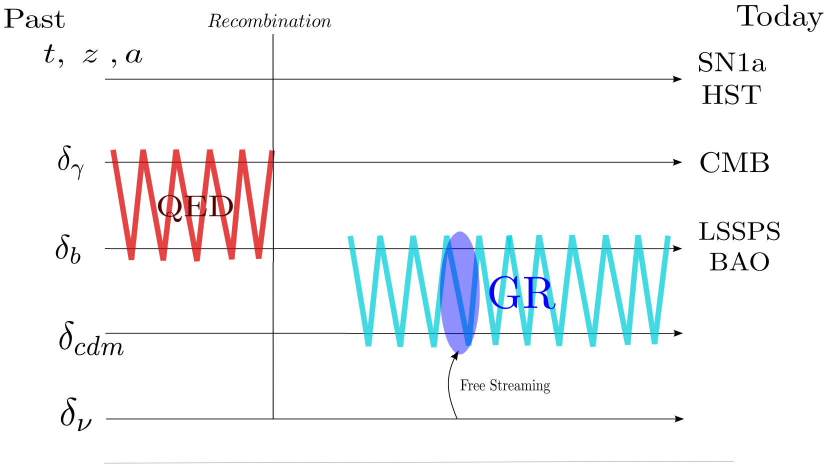
Introduction

- Cosmological linear theory



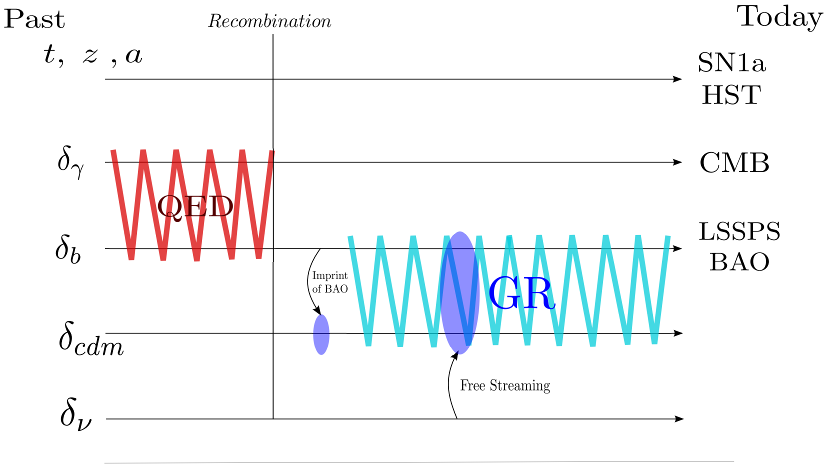
Introduction

- Cosmological linear theory



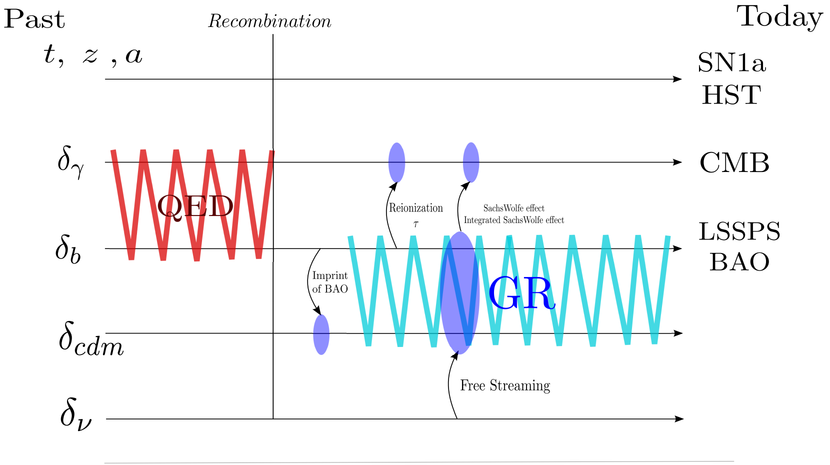
Introduction

- Cosmological linear theory



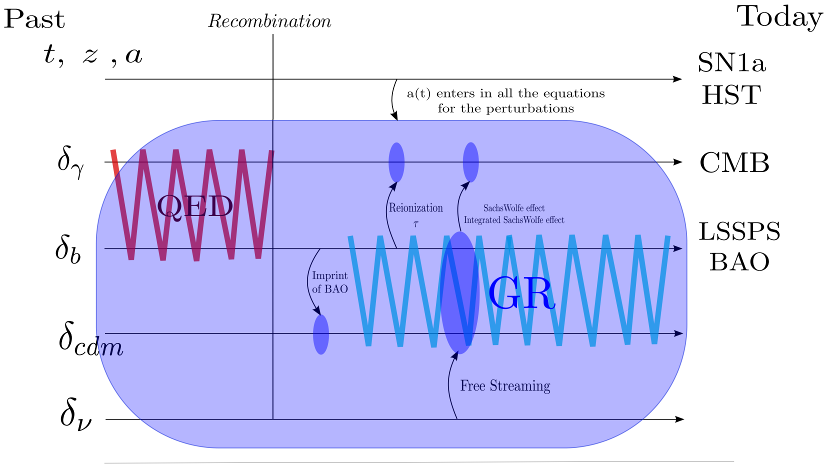
Introduction

- Cosmological linear theory



Introduction

- Cosmological linear theory



Introduction

- 0-order(homogeneous and isotropic),($\Omega_i \equiv \rho_i/\rho_{crit}, \;\; \rho_{crit} = \frac{3H^2}{8\pi G}$)
  - Matter $\rightarrow \Omega_m \rightarrow \Omega_{cdm}, \; \Omega_b$
  - Radiation $\rightarrow \Omega_r \rightarrow \Omega_\gamma$ (fixed by T_{CMB}), N_{rel}
  - Reionization optical depth $\rightarrow \tau$
  - Hubble parameter today $\rightarrow H_0 \rightarrow \Omega_\Lambda$
- 1-order, initial conditions for $\delta\rho/\rho$ are determined by the primordial power spectrum from inflation,
  - Primordial spectrum amplitude $\rightarrow A_s$
  - Spectral index($n_s = 1 \Rightarrow$ flat spectra) $\rightarrow n_s$

$$P(k) = A_s \frac{k^{1-n_s}}{k^3} \;\; \rightarrow \;\; C_l, P_{gal}(k)$$

Introduction

Why do we care about cosmological measurements of G_N?

- In general the gravitational constant at large scales need not be the same as the local value.
- Constraints from cosmological data will serve as an independent measurement of G_N at these large length scales.
- Want to learn more about dark matter and constrain its gravitational constant.

Table of Contents

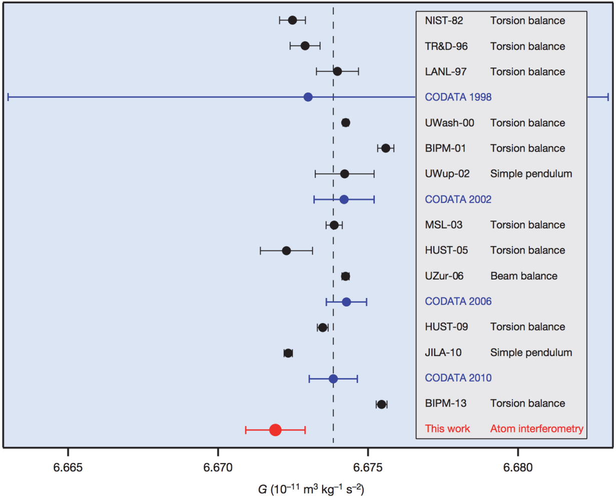
How can we measure the gravitational constant G_N?

- It is well-known that the gravitational acceleration of a probing body of mass m depends only on the product of Newton's Constant G_N and the central body mass M.

$$a_{\mathrm{grav}} = -\frac{G_N M}{r^2}$$

- To break this degeneracy and measure G_N, an additional force is required to define the central body mass.
- A variety of methods have been employed, both terrestrial and cosmological in origin.
- Current standard is $G_N = 6.67384(80) \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$ from CODATA 2010 with a relative error of 1.2×10^{-4}.

Terrestrial Measurements of G_N



G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli and G. M. Tino, *Nature* **510** (2014)

Cosmological Measurements of G_N

- Existing studies in the literature have used data from the primordial abundances of light elements synthesized by BBN and cosmic microwave background (CMB) anisotropies to constraint G_N, as well as other fundamental constants.
- K.-i. Umezu, K. Ichiki, and M. Yahiro, Phys.Rev. **D72**, (2005) constrained deviations of G_N at the $\sim 5\%$ level using BBN.
- S. Galli, A. Melchiorri, G. F. Smoot, and O. Zahn, Phys.Rev. **D80** (2009), provided a similar constraint using WMAP+BBN data.
- In this work we use the latest available cosmological data to update the constraint on G_N.

Cosmology with a Modified Gravitational Constant

- We introduce a dimensionless parameter λ_G to quantify deviations of the gravitational constant from G_N (as measured in Earth based laboratory experiments)

$$G = \lambda_G^2 G_N$$

- The introduction of λ_G modifies the Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} a^2 \lambda_G^2 G_N \bar{\rho}$$

*Dots indicate derivatives with respect to conformal time τ.

Invariance of the Background Evolution

But does this modification to the Friedmann equation actually have any physical consequences?

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} a^2 \lambda_G^2 G_N \bar{\rho}$$

No, we can just rescale time

$$\tau \to \lambda_G \tau$$

and the Friedmann equation is invariant. The background evolution (zero order) is degenerate with the "expansion clock".

First Order Fluid Perturbations

Energy-Momentum Conservation (Hydrodynamical Equations)

$$T^{\mu\nu}{}_{;\mu} = \partial_\mu T^{\mu\nu} + \Gamma^\nu{}_{\alpha\beta} T^{\alpha\beta} + \Gamma^\alpha{}_{\alpha\beta} T^{\nu\beta} = 0$$

For pressureless, non-interacting baryons the first order perturbations to the hydrodynamical equations are (in the Conformal Newtonian gauge)

$$\dot{\delta}_b = -\theta_b + 3\dot{\phi}$$

$$\dot{\theta}_b = -\frac{\dot{a}}{a}\theta_b + k^2\psi$$

Here, $\delta_b \equiv \delta\rho_b/\bar{\rho}_b$, $\theta_b \equiv ik_j v_b^j$, and $ds^2 = a^2(\tau)\{-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i\}$.

Cosmology with a Modified Gravitational Constant

If we re-scale time in the Friedmann equation

$$H^2 = \left(\frac{a'}{a}\right)^2 = \frac{8\pi}{3} a^2 \, G_N \bar{\rho}$$

where primes indicate derivatives with respect to $\tau' = \lambda_G \tau$, the parameter λ_G will be introduced into the first order perturbation equations

$$\lambda_G \delta_b' = -\theta_b + 3\lambda_G \phi'$$

$$\lambda_G \theta_b' = -\lambda_G \frac{a'}{a} \theta_b + c_s^2 k^2 \delta_b + k^2 \psi$$

Since $\theta_b = i k_j v_b^j$, if we rescale the wavenumbers by $k \to k/\lambda_G$ the first order perturbation equations are also invariant.

Wavenumber Rescaling

The rescaling of wavenumbers does NOT lead to an observable change because looking at the primordial power spectrum

$$P_s(k) = A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s - 1},$$

we see that a rescaling of the wavenumbers is degenerate with the amplitude of power spectrum A_s, a free parameter in the ΛCDM concordance model.

- The effect of changing G is to cause the universe to expand faster or slower by a factor of λ_G.
- Since gravity has no preferred scale, this change is unobservable.
- An independent measure of the expansion rate is needed to make λ_G physical.

How can we use cosmology to constrain G_N?

In reality, the baryons interact electromagnetically with the photons. We need to add a Thomson scattering term to the hydrodynamical equations

$$\dot{\delta}_b = -\theta_b + 3\dot{\phi}$$

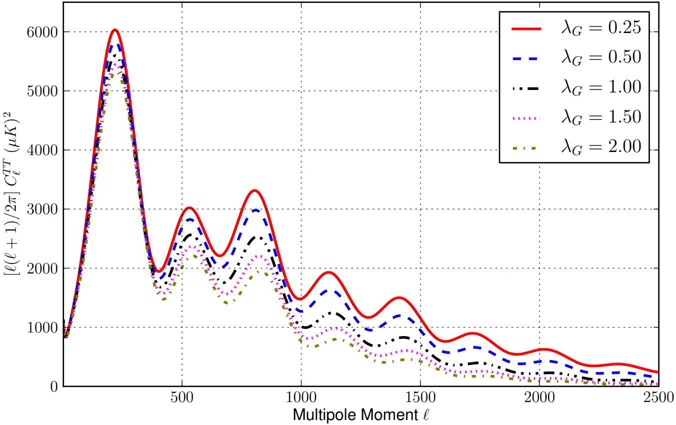
$$\dot{\theta}_b = -\frac{\dot{a}}{a}\theta_b + c_s^2 k^2 \delta_b + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b} a n_e \sigma_T (\theta_\gamma - \theta_b) + k^2 \psi$$

- Equations are no longer invariant under $\tau \to \lambda_G \tau$ and $k \to k/\lambda_G$.
- Thomson scattering term breaks the degeneracy by providing an independent measure of the expansion rate.
- Needed an interaction other than gravity to do this!
- Varying G now yields an observable change in cosmological evolution.

Table of Contents

CMB Temperature Power Spectrum

- Cosmological equations integrated and CMB spectra computed using the publicly available CLASS code.



Ionization Fraction

- If λ_G is increased (decreased), recombination takes place over a longer (shorter) period of time.

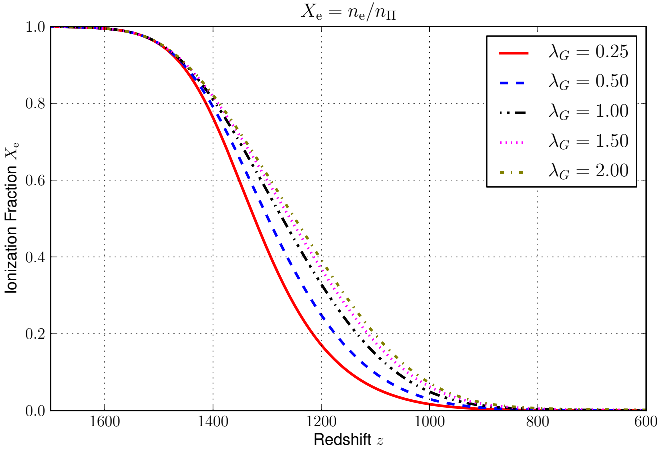


Table of Contents

Analysis Method

- Markov Chain Monte Carlo (MCMC) using the publicly available MontePython code (written to work with CLASS).
- For a given point in parameter space θ_i, compute observables using our modified CLASS code.
- Obtain $\mathcal{L}(D|\theta_i)$ using the package provided by the Planck collaboration which compares the output of the CLASS computation to the data.

$$P(\theta_i|D) = \frac{\mathcal{L}(D|\theta_i)\pi(\theta_i)}{\int \mathcal{L}(D|\theta_i) d\theta_1..d\theta_N} \tag{1}$$

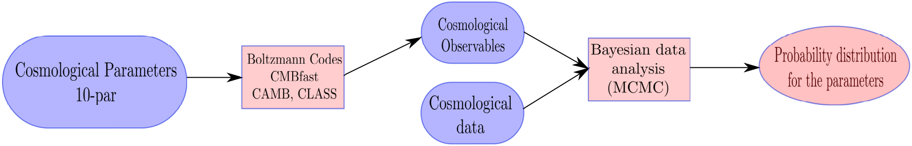
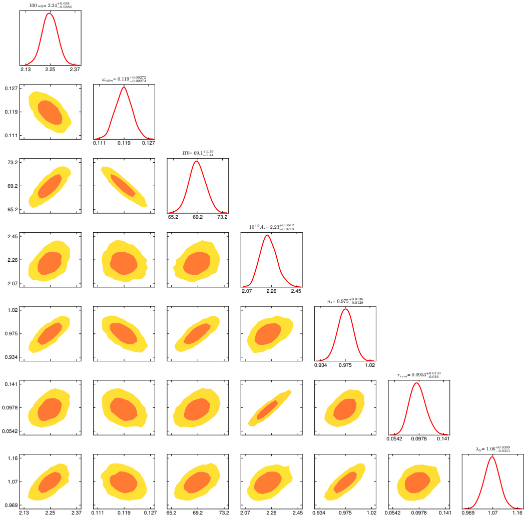


Table of Contents

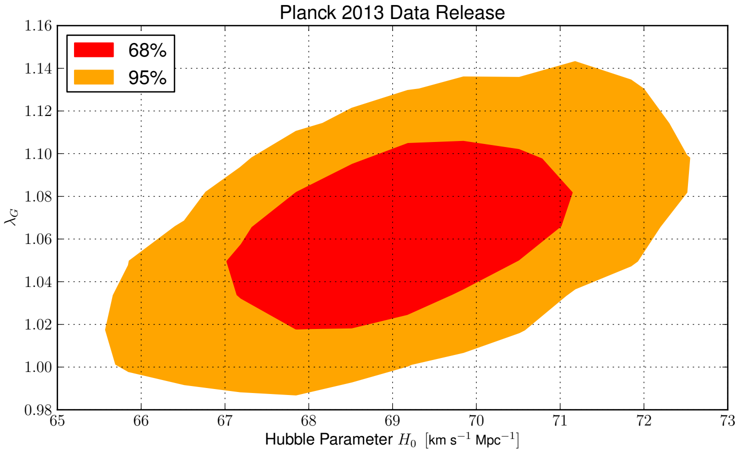
Experimental Data

- Planck 2013 Data Release (includes lensing likelihoods)
- 3 Yr, High-ℓ TT polarization from the Atacama Cosmology Telescope (ACT) and the South Pole Telescope (SPT).
  - ACT: 600 sq. deg. of sky at 148 and 218 GHz
  - SPT 800 sq. deg. of sky at 95, 150, 220 GHz
  - Combined, they cover $500 < \ell < 3500$
- BAO data from Sloan Digital Sky Survey (SDSS) (Data Releases 7 and 9) and Six degree Field Galaxy Survey (6dFGS).
  - SDSS Release 7 (9) covers 11,663 (14,555) sq. deg. of sky
  - 6dFGS covers $\sim 17,000$ sq. dg. of sky
  - Together, they cover a mean redshift range of 0.05-0.5
- H_0 measurement from Wide Field Camera 3 on HST $(0.01 < z < 0.1)$

Posterior Probability for the Parameters



Planck Constraint on λ_G



Results

Data	λ_G
Planck	$1.062^{+0.0309}_{-0.0311}$
Planck+Lensing+BAO	$1.041^{+0.0244}_{-0.0272}$
Planck+Lensing+BAO+HST	$1.046^{+0.0257}_{-0.0269}$
Planck+ACT/SPT	$1.026^{+0.0128}_{-0.0142}$

- The Planck+ACT/SPT dataset provides the best constraint on λ_G with a relative error of 1.4%. Thus, we report the cosmological measurement of the gravitational constant as

$$G_N(\text{cosmological}) = \lambda_G^2\, G_N = 7.025^{+0.176}_{-0.193} \times 10^{-11}\,\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}\,.$$

- This value has a relative error of 2.7% and is consistent with the CODATA value at $\sim 1.8\sigma$.

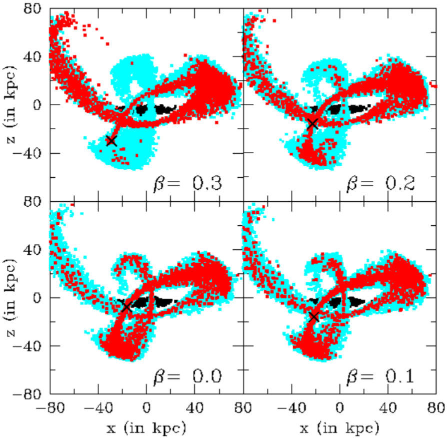
Table of Contents

Weak Equivalence Principle

- The Weak Equivalence Principle (WEP) states that all objects in a uniform gravitational field, independent of the mass or other compositional properties, will experience the same acceleration.
- In Newtonian language, the difference between inertial and gravitational mass must be exactly zero for the WEP to be respected.
- Modern experiments report that the difference between inertial and gravitational masses is zero at the 10^{-13} level. Thus, violations of the WEP in the visible sector are tightly constrained.
- However, WEP violation in the dark matter sector is far less constrained.

Constraints on WEP Violation for Dark Matter

- Kesden and Kamionkowski, *Phys. Rev. Lett.* **97** (2006), used the tidal disruption of the Sagittarius dwarf galaxy orbiting the Milky Way to constrain additional dark matter forces at the 10% level.



WEP Violation in the Dark Matter Sector

- We introduce WEP violation into the dark matter sector by allowing the gravitational charge of dark matter to differ from the inertial mass by a factor of λ_D
$$m_D^{\mathrm{grav}} = \lambda_D m_D$$

- Consequently, if we have two matter particles b_1 and b_2 and two dark matter particles D_1 and D_2, the gravitational forces in terms of the particle inertial masses are

$$F_{b_1,b_2} = -\frac{G_N m_{b_1} m_{b_2}}{r^2}, \quad F_{b_i,D_j} = -\lambda_D \frac{G_N m_{b_i} m_{D_j}}{r^2},$$

$$F_{D_1,D_2} = -\lambda_D^2 \frac{G_N m_{D_1} m_{D_2}}{r^2}.$$

General Coupled Friedmann Equations

We add in the other species by assuming they couple to gravity in the same way as the baryons

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{\mathcal{H}_0^2}{2}\left[\frac{\Omega_b}{a^3} + \frac{2\Omega_R}{a^4} + (1+3w)\frac{\Omega_\Lambda}{a^{3w+3}} + \frac{\lambda_D\Omega_D}{a_D^3}\right]$$

$$\frac{1}{a_D}\frac{d^2a_D}{dt^2} = -\frac{\mathcal{H}_0^2}{2}\left[\lambda_D\left(\frac{\Omega_b}{a^3} + \frac{2\Omega_R}{a^4} + (1+3w)\frac{\Omega_\Lambda}{a^{3w+3}}\right) + \frac{\lambda_D^2\Omega_D}{a_D^3}\right]$$

- No simple analytic integration to get first order Friedmann equations.
- We introduce the parameter $\mathcal{H}_0$ to distinguish from H_0, because for $\lambda_D \neq 1$, $\mathcal{H}_0$ is not the expansion rate today.
- For the usual ΛCDM cosmology, Ω_Λ is not an independent parameter (it is fixed by requiring a flat universe). For our case with two scale factors, we will keep Ω_Λ as a free parameter.

Two Fluid Decoupling: Initial Conditions

- Before the transition redshift z_T, we integrate the ordinary Friedmann equation, since everything evolves as a multi-component fluid described by a single scale factor $a_{\rm ord}$.
- After z_T, dark matter decouples and evolves as a separate fluid according to a dark scale factor a_D. The rest of the species evolve according to a scale factor a.
- Thus, our initial conditions are fixed by requiring $a_D = a = a_{\rm ord}$ and $\dot{a}_D = \dot{a} = \dot{a}_{\rm ord}$ at z_T.

Modified First Order DM Fluid Perturbations

- So far we have only considered modifications to the background evolution equations.
- When working in the $baryon$ co-moving frame, the dark matter fluid receives a modification to the first order perturbation equations.
- The first order perturbation equations for the other species stay the same.
- In what follows, dots indicate derivatives with respect to conformal time defined using ordinary baryon scale factor $dt = a(\tau)d\tau$.

Modified First Order DM Fluid Perturbations

The modified DM fluid perturbations in the baryon co-moving frame $\mathbf{x} = a(t)\,\mathbf{q}$ are

$$\dot{\delta}_D + \hat{\mathcal{D}}\delta_D + \theta_D = 0,$$

$$\dot{\theta}_D + (4H_D - 3H + 2\hat{\mathcal{D}})\theta_D + \nabla_q^2 \delta\psi = 0,$$

$$\nabla_q^2 \delta\psi = 4\pi G a^2 \left[\bar{\rho}_b \delta_b + \lambda_D \bar{\rho}_D \delta_D\right].$$

With the operator $\hat{\mathcal{D}}$ defined as follows

$$\hat{\mathcal{D}} = \left(1 - \frac{H}{H_D}\right)(\mathbf{v}_D^0 \cdot \nabla_q).$$

This operator $\hat{\mathcal{D}}$ is a directional derivative which translates from the dark matter frame to the baryon frame.

Modified First Order DM Fluid Perturbations

The modified first order dark matter fluid perturbations in k-space are

$$\dot{\delta}_D + (H - H_D)\,(3 + k\,\partial_k)\,\delta_D + \theta_D = 0,$$
$$\dot{\theta}_D + H\,\theta_D + 2\,(H - H_D)\,(1 + k\,\partial_k)\,\theta_D + k^2\,\delta\psi = 0,$$
$$k^2\delta\psi = 4\pi G a^2\left[\bar{\rho}_b\delta_b + \lambda_D\bar{\rho}_D\delta_D\right].$$

- Because the dark matter co-moving frame is not identical to the baryon one, bias terms proportional to $(H - H_D)$ enter the above equations.
- This frame conversion term contains k-derivatives which we implement using a finite difference method. This term couples adjacent modes.

Table of Contents

Effect of dark WEP breaking on the CMB TT Spectrum

$$\eta_D = \lambda_D - 1$$

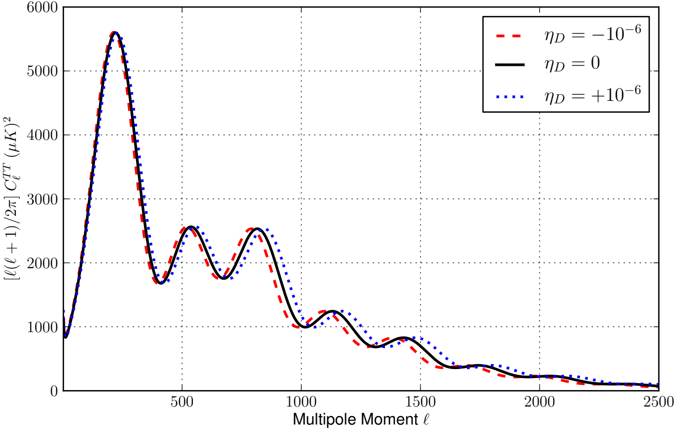
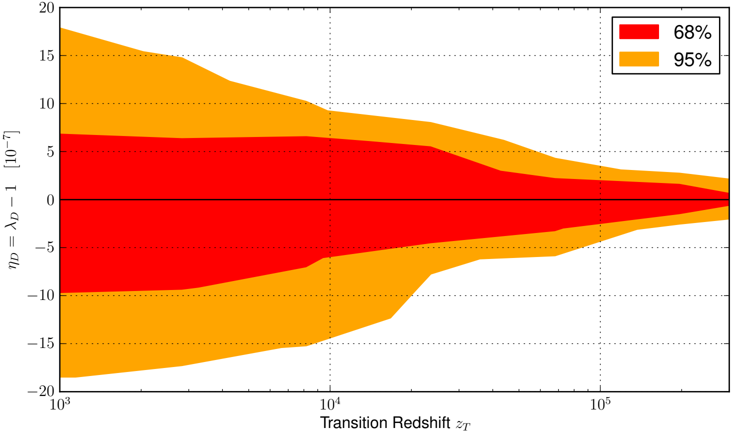


Table of Contents

Allowed region for λ_D as a function of z_T

- Using just data from Planck, $\lambda_D - 1$ is consistent with zero at the 10^{-6} level or less for all $z_T \geq 10^3$.



Tension in the Measurements of H_0

- Measurement of H_0 by Planck 2013

$$H_0 = 67.4 \pm 1.4 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

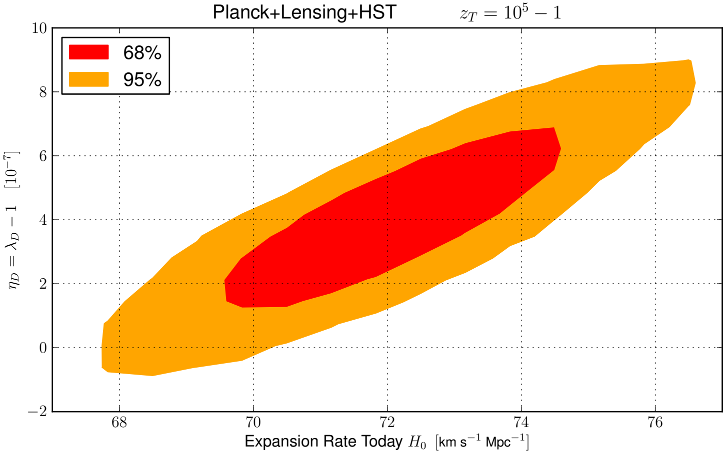
- Measurement of H_0 by Wide Field Camera 3 on HST

$$H_0 = 73.8 \pm 2.4 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

- Have $\sim 2\sigma$ tension between these measurements. Can our model help to alleviate this tension?

Hubble Space Telescope Prior

- Prefer $\lambda_D - 1 \neq 0$ at $\sim 2\sigma$ if we use data from the Hubble Space Telescope to impose a prior of $H_0 = 73.8 \pm 2.4 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}$.



Dark Energy Correlation with λ_D

- Although dark energy and λ_D have a similar effect at zero order, they are in fact independent parameters with non-trivial correlation.

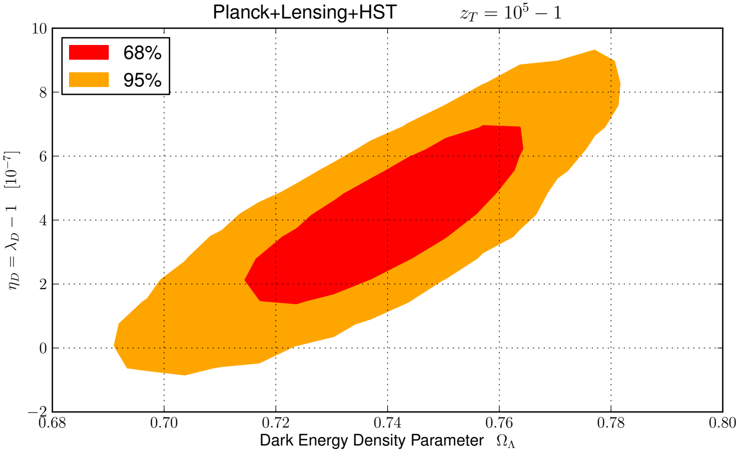


Table of Contents

Long Range Fifth Force Model

- Use a traditional long range "fifth force" to model different dynamics in the dark matter sector.

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\nabla^\mu\psi - m_\psi\bar{\psi}\psi - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + g\phi\bar{\psi}\psi$$

For scales smaller than $r_s = m_\phi^{-1}$, the Yukawa interaction mediates a fifth force. This fifth force will be long ranged if the mediator ϕ is nearly massless.

$$V(r) = -\frac{Gm_\psi^2}{r}\left[1 + \alpha_{\mathrm{Yuk}}\exp\left(-\frac{r}{r_s}\right)\right]$$

- Attempt to constrain α_{Yuk} using the latest cosmological data.

Rachel Bean, Eanna E. Flanagan, Istvan Laszlo, and Mark Trodden, **arXiv:0808.1105**, (2008)

Table of Contents

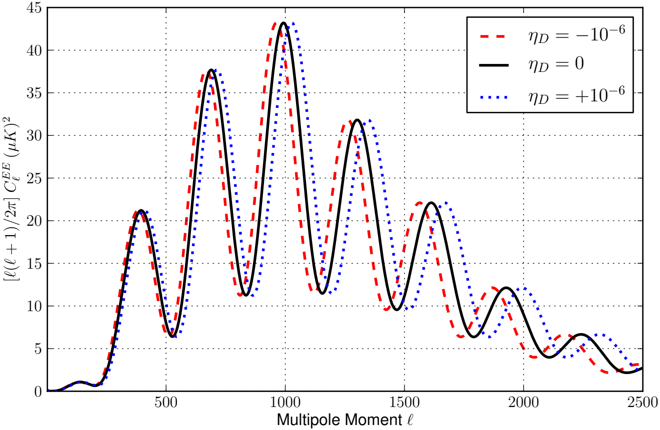
Conclusions

- We used the latest cosmological data to derive a constraint on G_N for all matter at the 2.7% level.
- We used a Newtoninan two fluid description to explicitly break the WEP in the dark matter sector.
- Using this method, we can constrain WEP in the dark matter sector at the 10^{-6} level or less for all $z_T \geq 10^3$.
- We intend to use the latest cosmological data to constrain a long range fifth force between dark matter particles.
- Cosmological data is very useful tool for studying the dark sector.

THE END

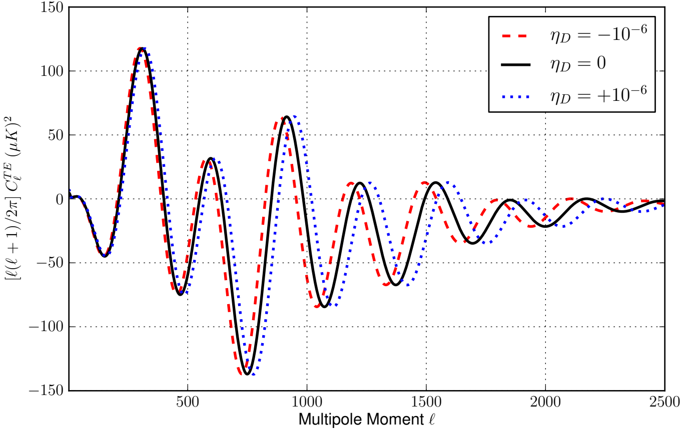
Effect of dark WEP breaking on the CMB EE Spectrum

$$\eta_D = \lambda_D - 1$$

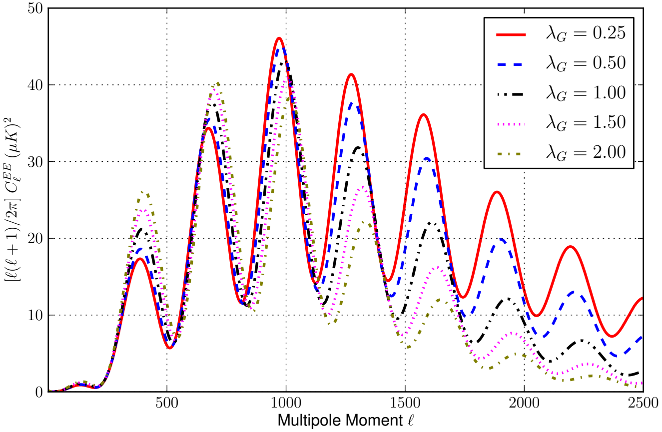


Effect of dark WEP breaking on the CMB TE Spectrum

$$\eta_D = \lambda_D - 1$$



CMB EE Power Spectrum



CMB TE Power Spectrum